Kernel Based Subspace Projection of Hyperspectral Images

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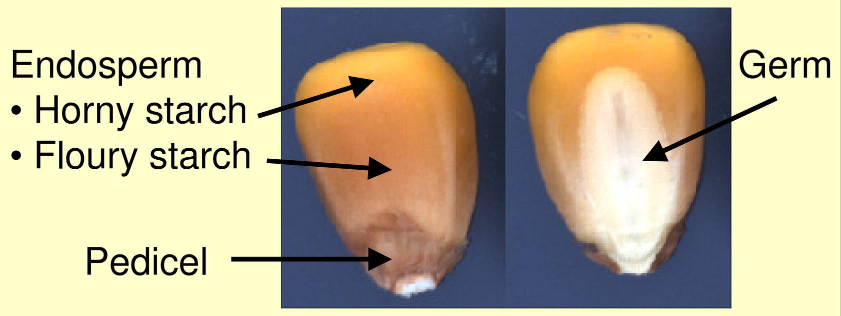
INTRODUCTION

In hyperspectral image analysis an exploratory approach to analyse the image data is to conduct subspace projections.

As linear projections often fail to capture the underlying structure of the data, we present kernel based subspace projections of PCA and Maximum Autocorrelation Factors (MAF). The MAF projection exploits the fact that interesting phenomena in images typically exhibit spatial autocorrelation.

The analysis is based on near-infrared hyperspectral images of maize grains demonstrating the superiority of the kernel-based MAF method.

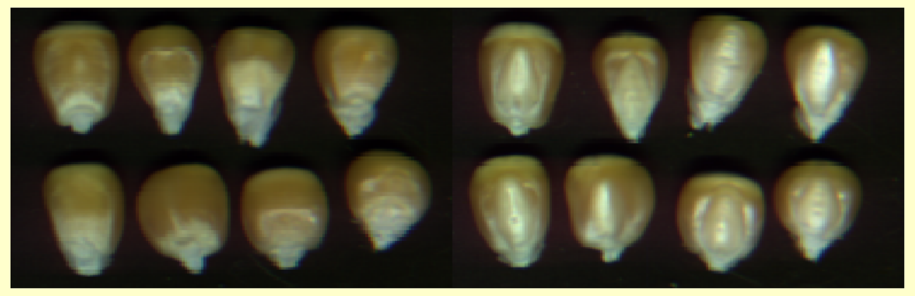
Maize grain kernel



HYPERSPECTRAL GRAIN DATA

A collection of 8 maize grains, front and backside, are used to generate a single hyperspectral image of *153* bands.

Pseudo RGB of maize grains.

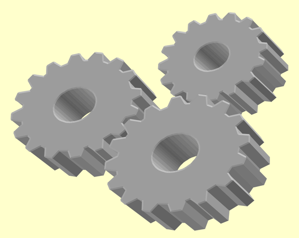


- Image size:
  370 x 149 x 153
- Spectral range:
  950 - 1700nm.

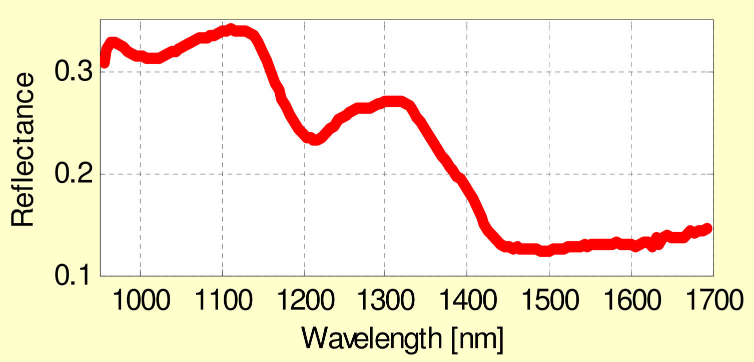
The hyperspectral image tensor is unfolded and represented as a n x p matrix X, where each row represents an observed pixel, i.e. X is a 55130 x 153 matrix.

Spectral Pre-Processing

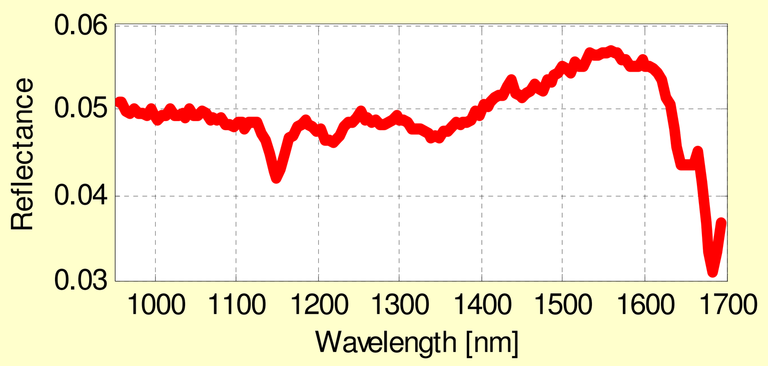
- Light source and dark current compensation.
- Remove 900-950nm (poor SNR).
- No spectral scatter correction.



Maize grain spectra (typical).



Background spectra (typical).



Subsampling

Appr. $\bar{n} = 3000$ random samples are used for extracting the projection vectors applied to all data pixels.

SUBSPACE PROJECTIONS

Linear Principal Component Analysis (PCA)

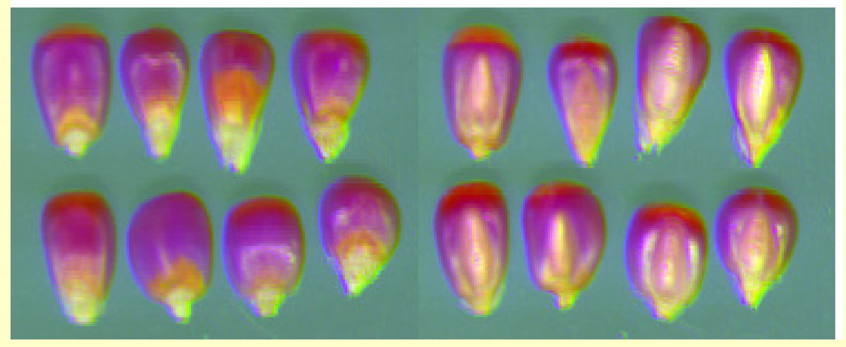
Eigenvalue problem formulation maximizing the variance

$$\Sigma u_i = \lambda_i u_i \qquad \text{, where} \quad \Sigma = \tfrac{1}{\bar{n}-1} X^T X$$

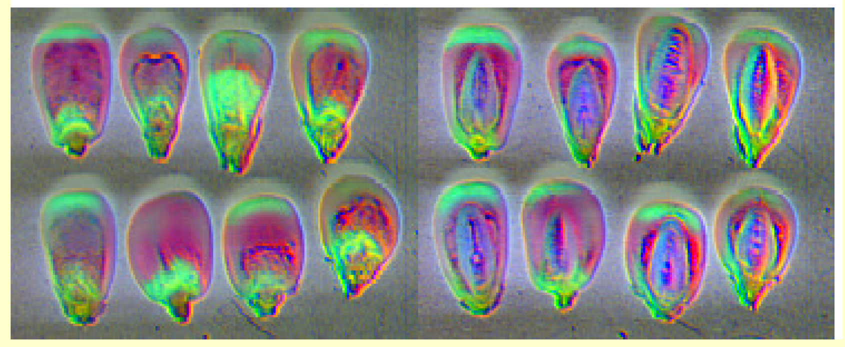
The orthonormal projection eigenvectors are expressed as $U = [u_1\ u_2 \ldots u_r]$ where $r = min(n,p)$. The subspace projection becomes $\bar{x} = U^T x$.

The dual formulation $\tfrac{1}{\bar{n}-1} X X^T v_i = \lambda_i v_i \;\Rightarrow\; u_i \propto X^T v_i \;\wedge\; v_i \propto X u_i$

Principal Components, PC1-PC3.



Principal Components, PC4-PC6.



Linear Maximum Autocorrelation Factor (MAF)

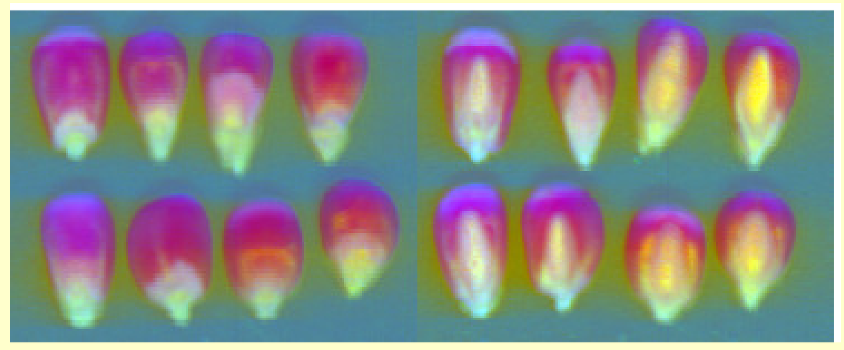
Maximise autocorrelation ρ of linear combinations $a^T x(r)$ of zero-mean spatial variables $x(r)$ at location r. The difference $x_\Delta(r) = x(r) - x(r+\Delta)$ has a covariance matrix Σ_Δ, where Δ is a displacement vector.

The autocorrelation ρ can be found as

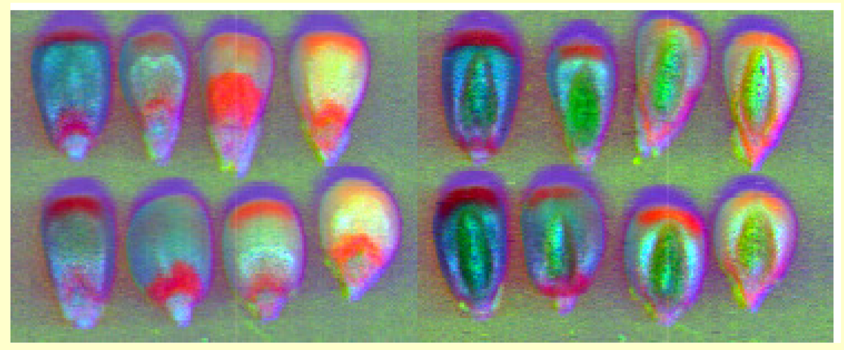
$$\rho = 1 - \frac{1}{2} \frac{a^T \Sigma_\Delta a}{a^T \Sigma a}$$

Properties
- Assumes 2nd order stationarity.
- Invariant to linear matrix transformation Tx_i, i.e. spectral scatter correction is not necessary.

MAF Components, MAF1-MAF3.



MAF Components, MAF4-MAF6.



Kernel PCA

Applying the *kernel trick* consist of mapping x_i into a higher dimensional feature space via the non-linear function $\phi(x)$, i.e. $x_i \rightarrow \phi(x_i)$.

The eigenvalue problem becomes

$$K v_i = \lambda_i v_i \qquad \text{, where} \quad K = \tfrac{1}{\bar{n}-1} \Phi \Phi^T \quad \wedge \quad \Phi = [\phi(x_1)^T\ \phi(x_2)^T \ldots \phi(x_{\bar{n}})^T]^T$$

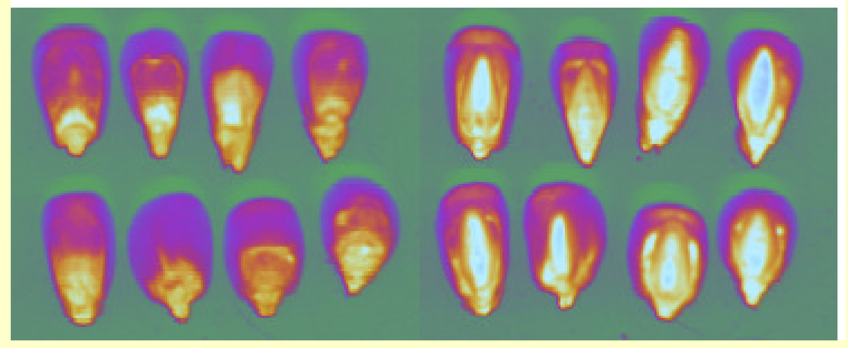
By exploiting the dual formulation the subspace projection can be found as

$$\Phi U = K V \Lambda^{-1/2}$$

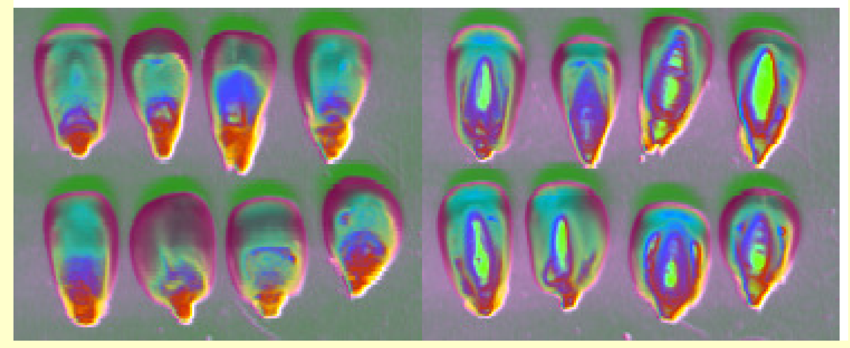
Projection is *memory-based* due to $K = [k(x, x_1)\ k(x, x_2) \ldots k(x, x_{\bar{n}})]$ leading to dependence on training dataset, i.e. partly non-parametric approach.

Gaussian kernel is given by $k(x_i, x_j) = \exp(-\tfrac{1}{2\sigma^2} \| x_i - x_j \|^2)$, where σ is set to the mean distance between the training observations.

Principal Components, PC1-PC3.



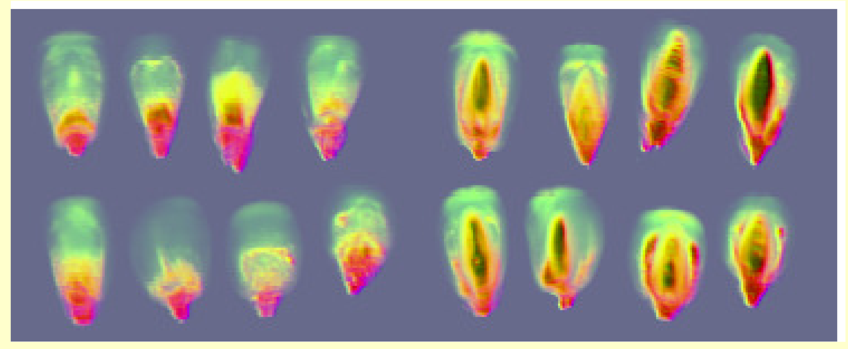
Principal Components, PC4-PC6.



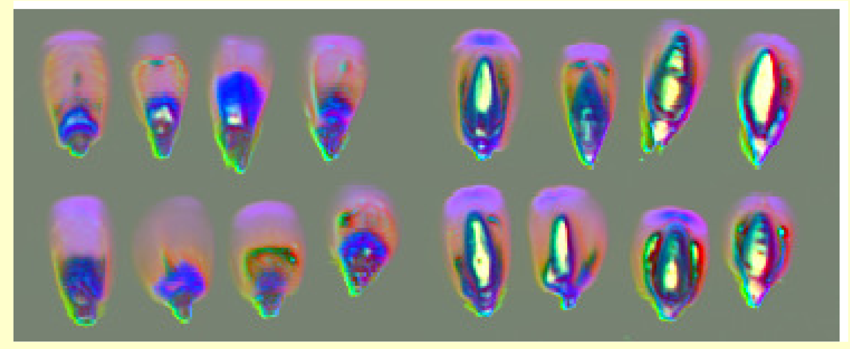
Kernel MAF

As for kernel PCA a similar framework can be derived for the kernel MAF method using the same Gaussian kernel.

MAF Components, MAF1-MAF3.



MAF Components, MAF4-MAF6.



CONCLUSION & REFERENCES

Conclusion

We have demonstrated how the kernel-based projections outperform the linear variants by their ability to suppress background noise, illumination and shadow effects.

The kernel MAF transform further provides a superior projection in terms of labelling different maize kernel parts with the same colour.

References

[1] R. Larsen, M. Arngren, P. W. Hansen and A. A. Nielsen, "*Kernel based subspace projection of near infrared hyperspectral images of maize kernels*", SCIA 2009.

[2] A. A. Nielsen, "*Kernel minimum noise fraction transformation*", submitted (2008).